

Chapter 1 Functions for Business Models1.1 Functions:- Domain and Range

A Mathematical model is an equation or a set of equations involving independent and dependent variables. An example of a mathematical model is a function.

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A function is a relationship between a dependent variable and one or more independent variables.

Usually written in the form:-

$$y = f(x, y, \dots)$$

Example:-

$$y = 2x + 3z$$

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Characteristics of a function

- ① For every set of independent variables on the domain, there can only be one corresponding dependent value.
- ② However, for every dependent value on the range, there may be one or more corresponding set of independent variable values.

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- ③ The set of all values an independent variable can take is the domain of the function.
- ④ The set of all values the dependent variable can generate is the range of the function.

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Example $y = x^2$

Here, x is the independent variable
and y is the dependent variable.

Because we get to choose what we assign into x (we get to choose only what's on the domain) and what y returns depends on what we assign into x .

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How do we call a function?

"The dependent variable is a function in terms of the independent variables."

Example $y = x^2$

" y is a function in terms of x ."

OR " y is a function of x ."

OR " y is in terms of x ."

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Another example:-

$$f(x) = 3x + \frac{1}{x}$$

$f(x)$ means f is a function in terms of the variable x .

Important:- $f(x)$ does NOT mean

f multiplies x

f is the dependent variable
and x is the independent variable.

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What is the domain of
 $f(x) = 3x + \frac{1}{x}$?

Answer:-

We cannot have a division by zero.

So we cannot have x to be zero

i.e. $x \neq 0$

$\therefore$ The domain is any number except 0

Domain is $x \in (-\infty, 0) \cup (0, \infty)$ OR $x < 0$ or $x > 0$

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Let's revise:-

① $a \leq x \leq b \Leftrightarrow x \in [a, b]$

Square brackets
mean the
interval includes
the endpoint(s)

② $a < x \leq b \Leftrightarrow x \in (a, b]$

③ $a \leq x < b \Leftrightarrow x \in [a, b)$

Round brackets
mean the interval
does not include
the endpoint(s)

④ $a < x < b \Leftrightarrow x \in (a, b)$

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Infinity is something we can never reach
so $-\infty < x < \infty \Leftrightarrow x \in (-\infty, \infty)$

Another example:-

$$y = \sqrt{x}$$

An argument of an even root must be non-negative

so we need $x \geq 0 \Leftrightarrow x \in [0, \infty)$

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Exercise

⑦ $y = \frac{1+x}{1-x}$. What's the domain?

Answer:- we need the denominator to be non-zero.

so we need

$$1-x \neq 0$$

$$\Rightarrow x \neq 1$$

$\therefore$ Domain is $x < 1$ or $x > 1$

OR $x \in (-\infty, 1) \cup (1, \infty)$

When $x = -1$,
 $y = \frac{1+(-1)}{1-(-1)} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

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⑧ $y = \frac{\sqrt{x-8}}{x-10}$. What's the domain?

Answer:-

we need $x-8 \geq 0$ and $x-10 \neq 0$

$$\Rightarrow x \geq 8 \text{ and } x \neq 10$$

$\therefore$ The domain is

$$8 \leq x < 10 \text{ or } x > 10$$

$$\text{OR } x \in [8, 10) \cup (10, \infty)$$

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